

Mathematics Calculation Policy

This policy has been designed to teach children through the use of concrete, pictorial and abstract methods. This calculation policy should be used to support children to develop a deep understanding of number and calculation.

This policy has been developed with an emphasis on Singapore methods to develop number awareness and fluency. The policy only details the strategies; teachers must plan opportunities for pupils to apply these; for example, when solving problems or where opportunities emerge elsewhere in the curriculum.

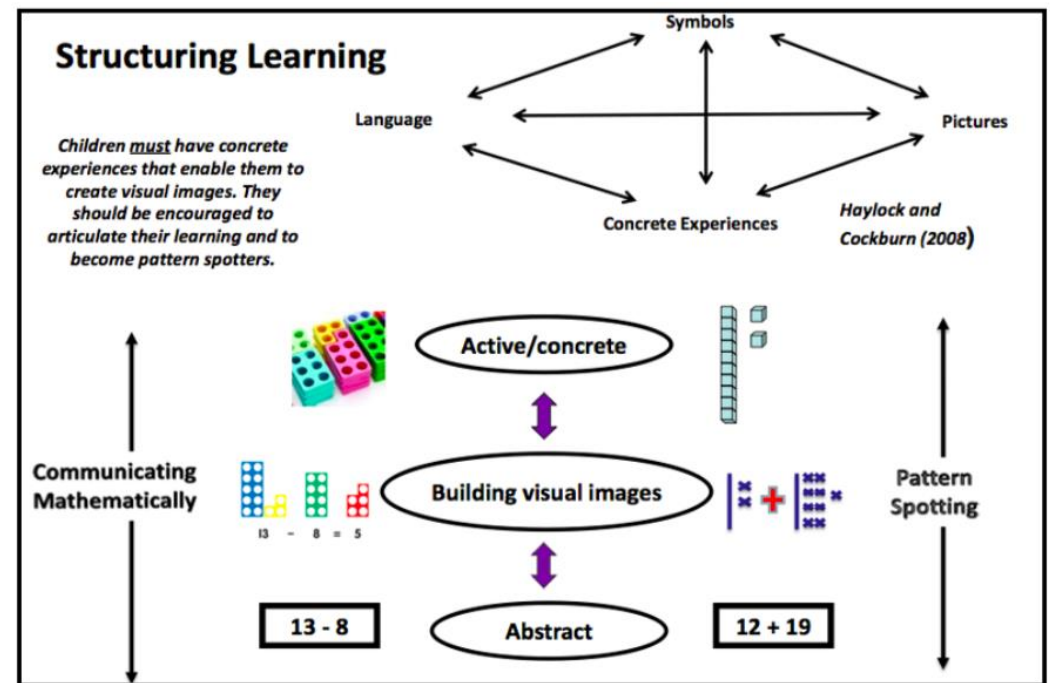
Using the concrete-pictorial-abstract approach:

Children develop an understanding of a mathematical concept through the three steps (or representation) of concrete-pictorial-abstract approach. Reinforcement is achieved by going back and forth between these representations.

Concrete representation The enactive stage - a pupil first introduced to an idea or a skill by acting it out with real objects. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

Pictorial representation The iconic stage - a pupil has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem.

Abstract representation The symbolic stage - a pupil now capable of representing problems by using mathematical notation, for example: $12 \div 2 = 6$.



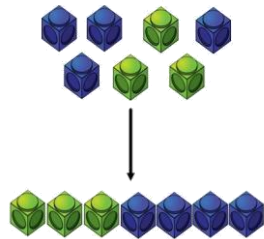
EYFS and Year 1

Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

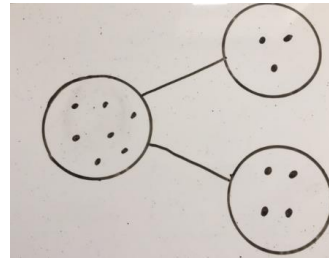
Concrete

Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars).



Pictorial

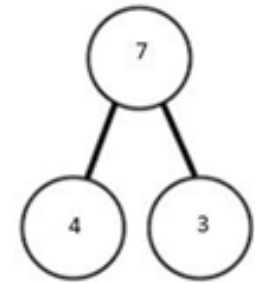
Children to represent the cubes using dots or crosses. They could put each part on a part-whole model too.



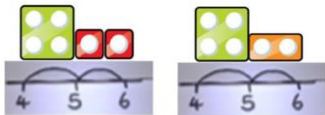
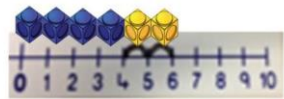
Abstract

$$4 + 3 = 7$$

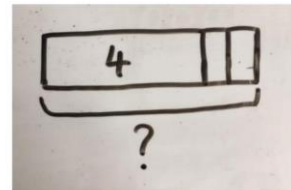
Four is a part, 3 is a part and the whole is seven.



Counting on using number lines using cubes or Numicon e.g. $4 + 2$



A bar model which encourages the children to count on, rather than count all.



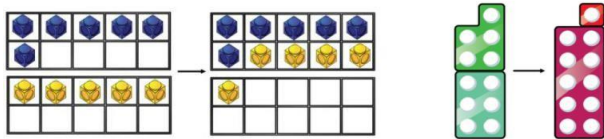
The abstract number line:

What is 2 more than 4?

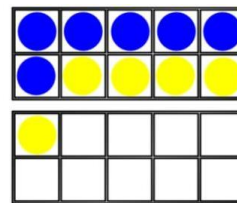
What is the sum of 2 and 4? What is the total of 4 and 2? $4 + 2$



Regrouping to make 10; using ten frames and counters/cubes or using Numicon. $6 + 5$



Children to draw the ten frame and counters/cubes.



Children to develop an understanding of equality e.g.

$$6 + \square = 11$$

$$6 + 5 = 5 + \square$$

$$6 + 5 = \square + 4$$

Year 2

Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

Concrete

Combining three parts/numbers/groups of objects using physical objects.

$$7 + 3 + 2$$

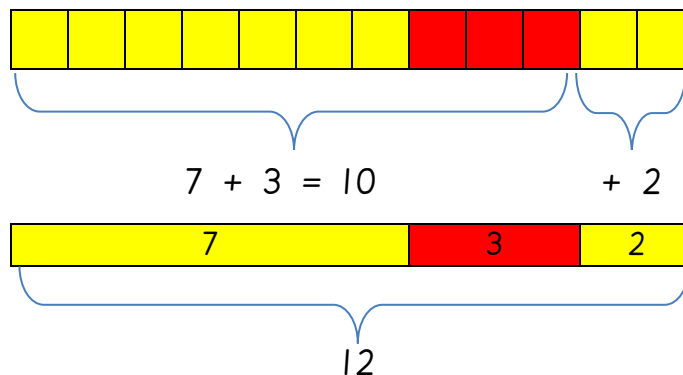


(Using knowledge of number bonds to 10 would lead to $10 + 2$)



Pictorial

Children to represent objects as part of a bar model.



Abstract

Recognition of number bonds and knowing to start with the larger number should result in children being able to carry out these number calculations written as number sentences.

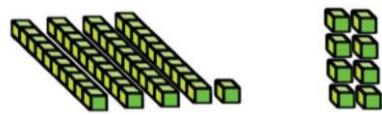
$$7 + 3 + 2$$

$$7 + 3 = 10$$

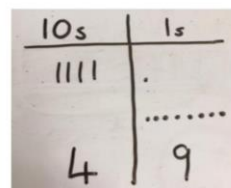
$$10 + 2 = 12$$

TO + O using base 10. Continue to develop understanding of partitioning and place value.

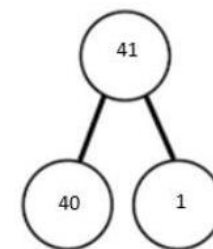
$$41 + 8$$



Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.

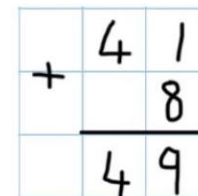


$$41 + 8$$



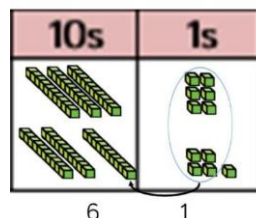
$$1 + 8 = 9$$

$$40 + 9 = 49$$

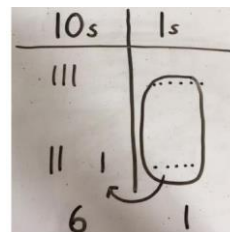


TO + TO using base 10. Continue to develop understanding of partitioning and place value.

$$36 + 25$$



Children to represent the base 10 in a place value chart.



Looking for ways to make 10.

$$36 + 25 =$$

$$1 \quad 5$$

$$30 + 20 = 50$$

$$5 + 5 = 10$$

$$50 + 10 + 1 = 61$$

Formal written method

$$\begin{array}{r} 36 \\ +25 \\ \hline 61 \\ 1 \end{array}$$

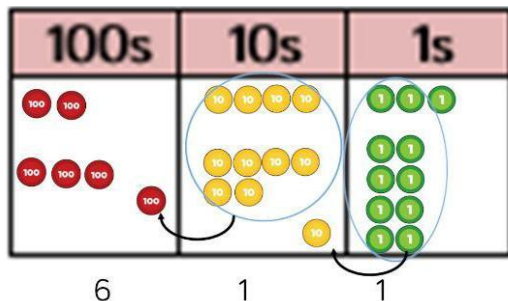
Years 3, 4, 5 and 6

Addition

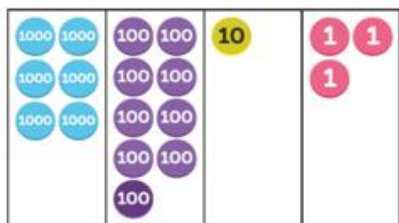
Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

Concrete

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten. When there are 10 tens in the 10s column- we exchange for 1 hundred. e.g. $243 + 368$



This method would be extended to adding four digit numbers in Year 4 e.g. $5678 + 1235$



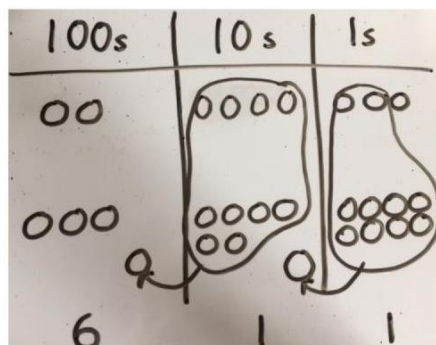
(This shows how the chart would look after each column has been added and exchanges have happened.)

In Year 5, the chart would be extended to five and six digit numbers including decimals.

Columns to allow numbers up to 10 million and decimal numbers up to three decimal places would be used in Year 6.

Pictorial

Children to represent the counters in a place value chart, circling when they make an exchange.



This method again could be developed for Years 4, 5, and 6 by introducing more columns and eventually a decimal point. Children will need to draw their own place value chart with the number and format of the columns determined by the numbers that are being added.

Abstract

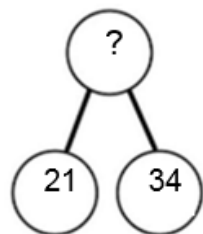
Formal written method

$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \text{1 1} \end{array}$$

	Th	H	T	O
	4	6	2	7
+	3	9	1	4
	8	5	4	1
	1		1	

	0	•	5		5		7
	1	•	2		1		1
+	0	•	2		0		2
	1	•	9		7		0
					1		

Addition Conceptual Variation; different ways to ask children to solve $21 + 34$



?	
21	34

Word problems:

In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?

$$21 + 34 = 55. \text{ Prove it}$$

$$21 + 34 =$$

21

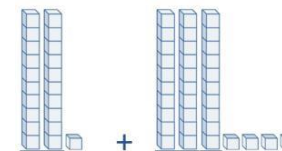
+34

—

$$21 + 34 =$$

$$\square = 21 + 34$$

Calculate the sum of twenty-one and thirty-four.



Missing digit problems:

10s	1s
10 10	1
10 10 10	?
?	5

Addition Problem Solving using the Bar Model

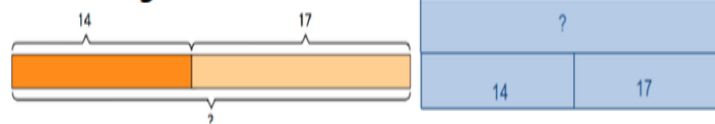
EYFS and Year 1

Sara has 2 apples.
Jon has 5 apples.
How many apples do they have altogether?
How many more apples does Jon have than Sara?



Year 2

Helen has 14 breadsticks. Her friend has 17. How many do they have altogether?



Year 3

A man sold 230 balloons at a carnival in the morning. He sold another 86 balloons in the evening. How many balloons did he sell in all?

?	
230	86
Morning	Afternoon

Year 4

Alison jogs 6,860 metres and Calvin jogs 5,470 metres. How far do they jog altogether?

?	
6860m	5470m

Year 5

McDonalds sold £957.68 worth of hamburgers and 1238.50 worth of chicken nuggets. How much money did they take altogether?

?	
£957.68	£1238.50

Year 6

Jack went on holiday. His flight cost £70.50, the hotel £1295 and spending money £427.89. How much did Jack spend on his holiday?

?		
£70.50	£427.89	£1295

EYFS, Year 1 and Year 2

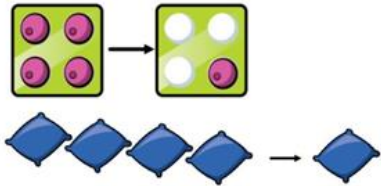
Subtraction

Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

Concrete

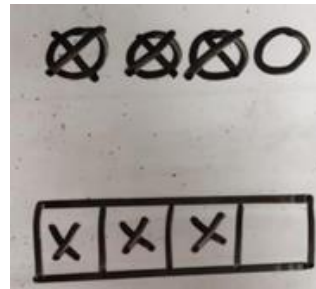
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).

$$4 - 3 = 1$$



Pictorial

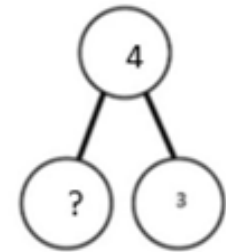
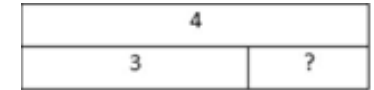
Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.



Abstract

$$4 - 3 =$$

$$\square = 4 - 3$$

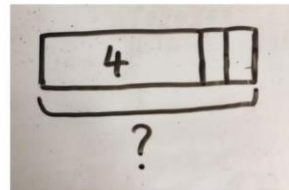


Counting back (using number lines or number tracks) children start with 6 and count back 2.

$$6 - 2 = 4$$



A bar model which encourages the children to count on, rather than count all.



The abstract number line:

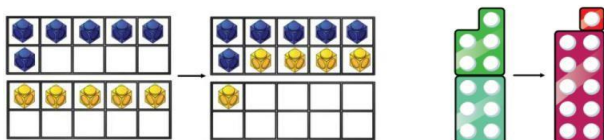
What is 2 more than 4?

What is the sum of 2 and 4? What is the total of 4 and 2? $4 + 2$

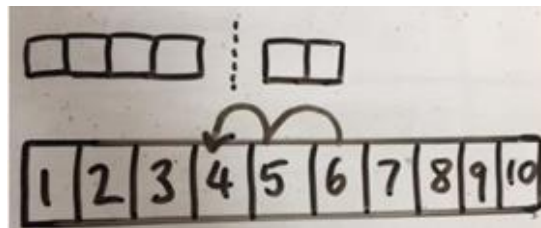


Regrouping to make 10; using ten frames and counters/cubes or using Numicon.

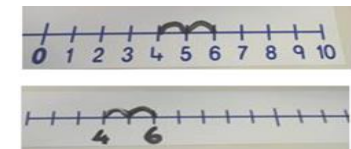
$$6 + 5$$



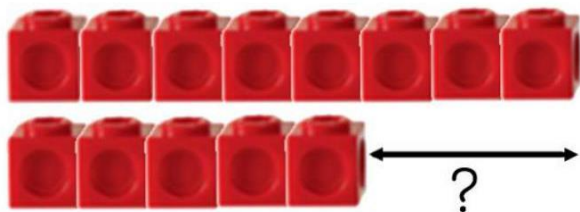
Children to represent what they see pictorially e.g.



Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line.

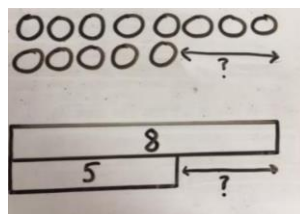


Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).
Calculate the difference between 8 and 5.



This is a physical representation of the part-whole/ bar model where 8 is the 'whole' and 5 is one of the 'parts'. To find the difference between 8 and 5, children need to work out what the other 'part' is.

Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.



$$76 - 23 =$$

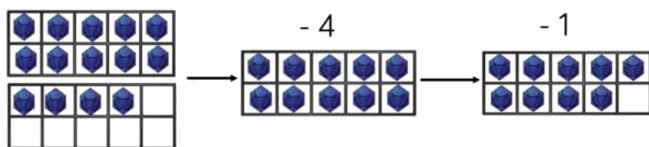
76	
23	?

Children should start to develop an understanding of the inverse relationship between addition and subtraction.

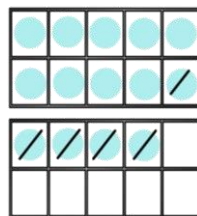
Find the difference between 8 and 5.
 $8 - 5$, the difference is $\square$

Children to explore why
 $9 - 6 = 8 - 5 = 7 - 4$ have the same difference.

Making 10 using ten frames.
 $14 - 5$



Children to present the ten frame pictorially and discuss what they did to make 10.



Children to show how they can make 10 by partitioning the subtrahend.

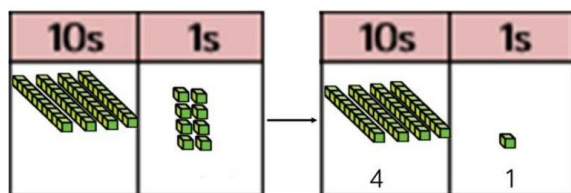
$$14 - 5 = 9$$

4 1

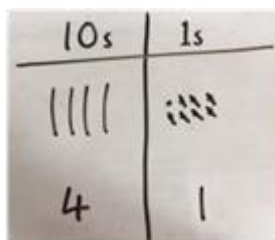
$$14 - 4 = 10$$

$$10 - 1 = 9$$

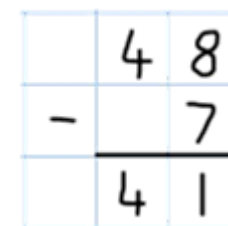
Column method using base 10.
 $48 - 7$



Children to represent the base 10 pictorially.



Column method or children could count back 7.



Years 3, 4, 5 and 6

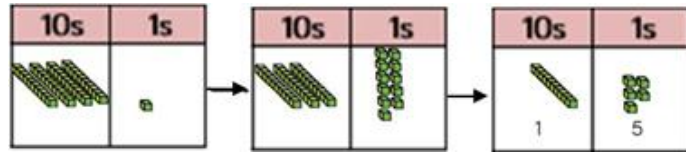
Subtraction

Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

Concrete

Column method using base 10 and having to exchange.

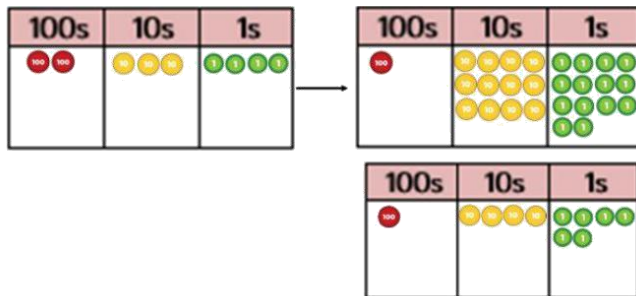
$$41 - 26$$



(In the second grid, one of the tens has been exchanged for 10 ones so that 26 can be physically taken away.)

Column method using place value counters.

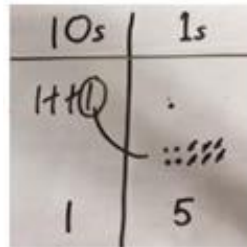
$$234 - 88$$



Pictorial

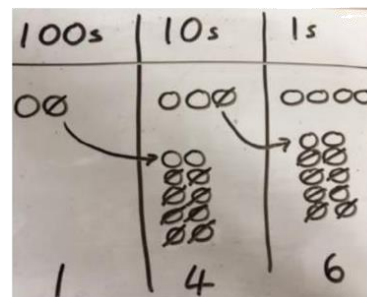
Represent the base 10 pictorially, remembering to show the exchange by circling.

$$41 - 26$$



Represent the place value counters pictorially; remembering to show what has been exchanged.

$$234 - 88$$



Abstract

Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$.

$$\begin{array}{r} 41 \\ - 26 \\ \hline 15 \end{array}$$

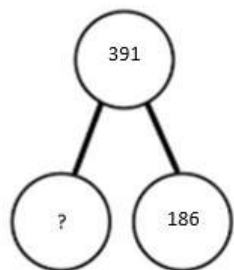
Formal column method. Children must understand what has happened when they have crossed out digits.

$$\begin{array}{r} 234 \\ - 156 \\ \hline 1192 \end{array}$$

It is important to use in a range of contexts- measures and money.

$$\begin{array}{r} 105.341 \text{ kg} \\ - 36.080 \text{ kg} \\ \hline 69.339 \text{ kg} \end{array}$$

Subtraction Conceptual Variation; different ways to ask children to solve $391 - 186$



391	
186	?

Raj spent £391, Timmy spent £186.

How much more did Raj spend?

Calculate the difference between 391 and 186.

$$\square = 391 - 186$$

$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

What is 186 less than 391?

Missing digit calculations:

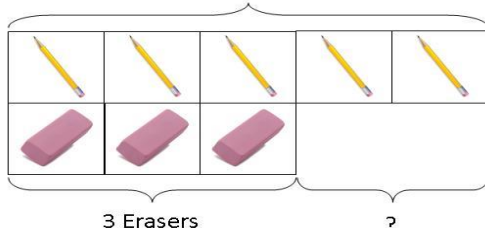
$$\begin{array}{r} 39\square \\ -\square\square6 \\ \hline \square05 \end{array}$$

Subtraction Problem Solving using the Bar Model

EYFS and Year 1

Peter has 5 pencils and 3 erasers. How many more pencils than erasers does he have?

5 Pencils



3 Erasers

?

Year 4

There are 3,160 books in a shop. 1,226 are in English and the rest are in French. How many French books are there?

3160	
1226	?

Year 2

Sarah picked 76 flowers. 23 of them were yellow, how many of them were pink?

76	
23	?

Year 5

A whole to Lapland costs £5005 for a family of four, the Smith's have only saved £3787.75, how much money do they still need to find?

£5005	
?	£3787.75

Year 3

315	$315 - 185 = ?$
185	$185 + ? = 315$

?	$185 + 315 = ?$
185	$? - 185 = 315$

Year 6

Chloe wants to buy a new car for £6450. She has £4885.87 in her savings account. Her Dad gives her £150 for her birthday. How much more money does she need to save?

£6450		
£4885.87	£150	?

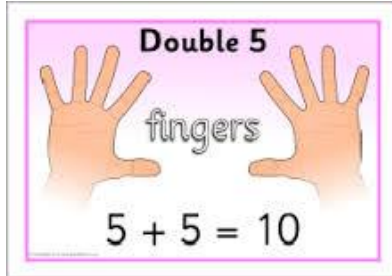
EYFS and Year 1

Multiplication

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Concrete

Making it clear you are adding the same number.



Pictorial



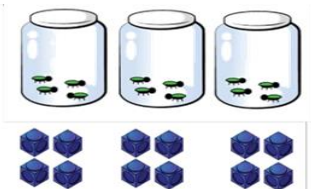
Abstract

$$2 + 2 = 4$$

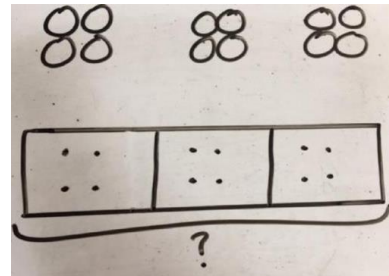
$$5 + 5 = 10$$

$$3 + 3 = 6$$

Repeated grouping/repeated addition
 3×4
 $4 + 4 + 4$
 There are 3 equal groups, with 4 in each group.



Children to represent the practical resources in a picture and use a bar model.



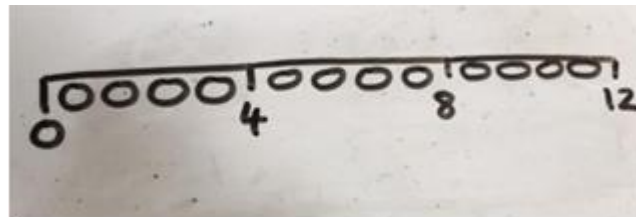
$$3 \times 4 = 12$$

$$4 + 4 + 4 = 12$$

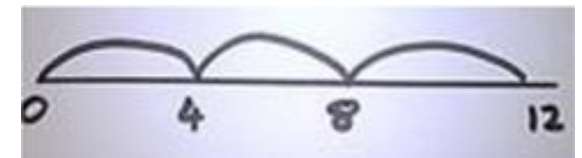
Number lines to show repeated groups
 3×4



Represent this pictorially alongside a number line e.g.:



Abstract number line showing three jumps of four.
 $3 \times 4 =$



Year 2 and 3

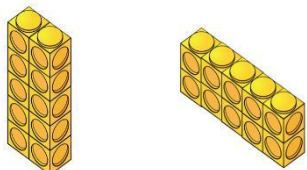
Multiplication

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Concrete

Use arrays to illustrate commutatively
counters and other objects can also be used.

$$2 \times 5 = 5 \times 2$$

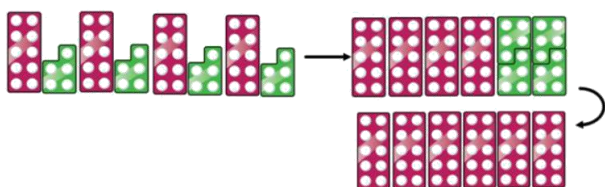


2 lots of 5

5 lots of 2

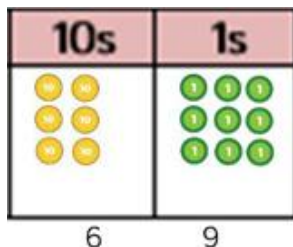
Partition to multiply using Numicon,
base 10 or Cuisenaire rods.

$$4 \times 15$$



Formal column method with place value
counters (base 10 can also be used.)

$$3 \times 23$$

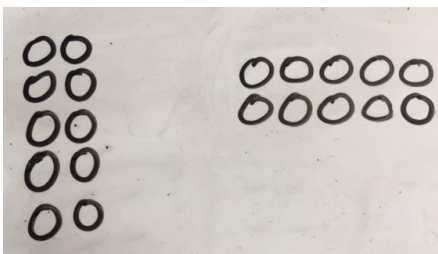


6

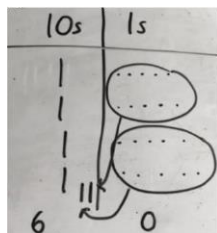
9

Pictorial

Children to represent the arrays
pictorially.



Children to represent the concrete
manipulative pictorially.



Abstract

Children to be able to use an array to
write a
range of calculations e.g.

$$10 = 2 \times 5$$

$$5 \times 2 = 10$$

$$2 + 2 + 2 + 2 + 2 = 10$$

$$10 = 5 + 5$$

Children to be encouraged to show the
steps they have taken.

$$4 \times 15$$

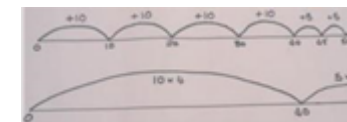
$$10 \quad 5$$

$$10 \times 4 = 40$$

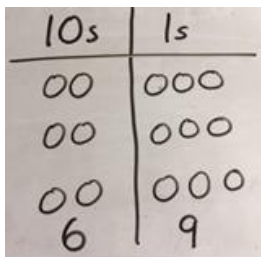
$$5 \times 4 = 20$$

$$40 + 20 = 60$$

A number line can
also be used:



Children to represent the counters
pictorially.



Children to record what it is they are
doing to show understanding.

$$3 \times 23$$

$$3 \times 20 = 60$$

$$3 \times 3 = 9$$

$$60 + 9 = 69$$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

Years 4, 5 and 6

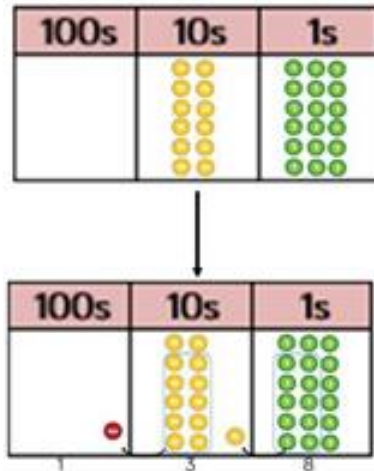
Multiplication

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Concrete

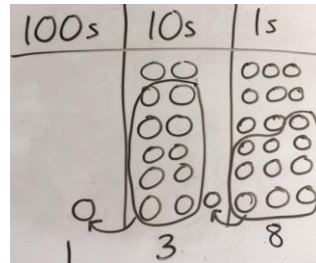
Formal column method with place value counters.

$$6 \times 23$$



Pictorial

Children to represent the counters/base 10, pictorially e.g. the image below.



Abstract

Formal written method:

$$6 \times 23 =$$

$$\begin{array}{r} 23 \\ \times 6 \\ \hline 138 \\ 11 \end{array}$$

$$\begin{array}{r} 124 \\ \times 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \\ 11 \end{array}$$

Answer: 3224

When children start to multiply 3 digits $\times$ 3 digits and 4 digits $\times$ 2 digits etc., they should be confident with the abstract:

To get 744 children have solved 6×124 .

To get 2480 they have solved 20×124 .

Multiplication Conceptual Variation; different ways to ask children to solve 6×23

23	23	23	23	23	23
----	----	----	----	----	----

?

Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that $6 \times 23 = 138$

Find the product of 6 and 23

$$6 \times 23 =$$

$$\square = 6 \times 23$$

$$\begin{array}{r} 6 \quad 23 \\ \times \quad 23 \\ \hline \end{array} \quad \begin{array}{r} 23 \\ \times 6 \\ \hline \end{array}$$

What is the calculation?
What is the product?

100s	10s	1s
		

Multiplication Problem Solving

EYFS and Year 1

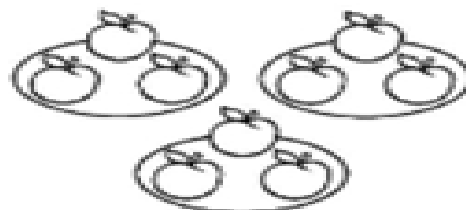
Children will experience equal groups of objects.

They will work on practical problem solving activities involving



There are 6 pairs of socks.
How many socks are there altogether?

Year 2



How many apples are there altogether?

$$3 + 3 + 3 = 9$$

Year 3

4 children go to the cinema.
They each pay £15. How much do they spend altogether?

?			
15	15	15	15

Year 4

A computer costs 5 times as much as a television. The television costs £429.

How much does the computer cost?

?				
£429				

Year 5

The cost to run a sports centre is £4375 a week, how much would it cost to run for 16 weeks?

?															

£4375
a week

Year 6

If 5 friends went on holiday and each paid £579.75 what was the total cost of the holiday?

Cost of the holiday

?				
£579.75				

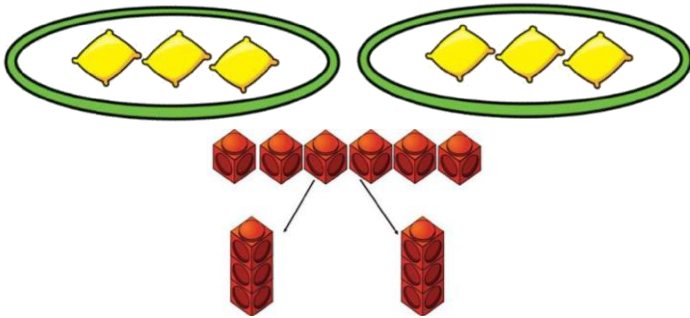
EYFS, Year 1 and Year 2

Division

Key language: share, group, divide, divided by, half.

Concrete

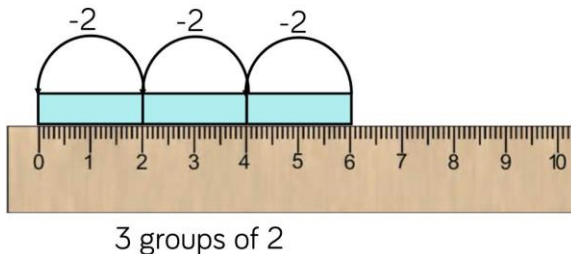
Sharing using a range of objects.
 $6 \div 2$



Children physically group items and count in groups.

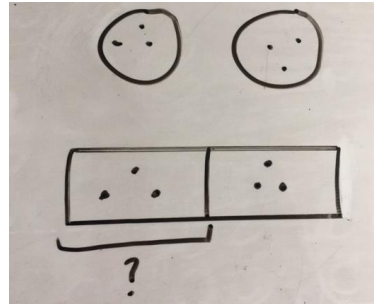


Repeated subtraction using Cuisenaire rods above a ruler.
 $6 \div 2$



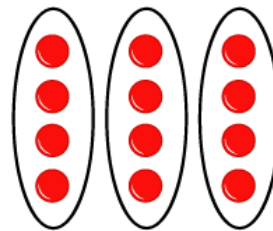
Pictorial

Represent the sharing pictorially.

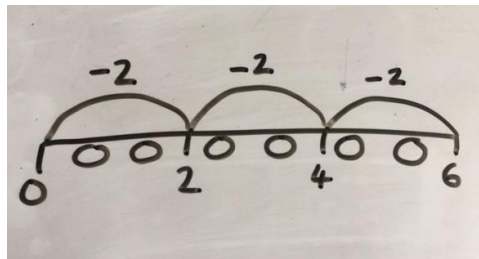


Model forming arrays to aid counting when this develops into counting in multiples.

$$12 \div 3 = 4$$

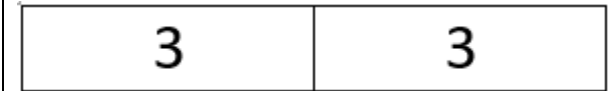


Children to represent repeated subtraction pictorially.



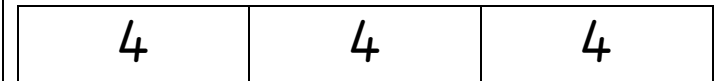
Abstract

$$6 \div 2 = 3$$

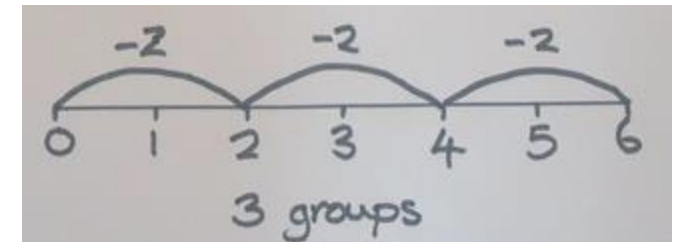


Children should also be encouraged to use their 2 times tables facts.

$$12 \div 3 = 4$$



Abstract number line to represent the equal groups that have been subtracted.



Year 3

Division

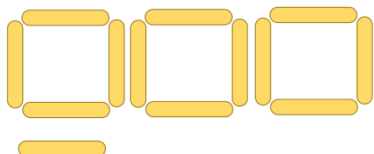
Key language: share, group, divide, divided by, half.

Concrete

2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used.

$$13 \div 4$$

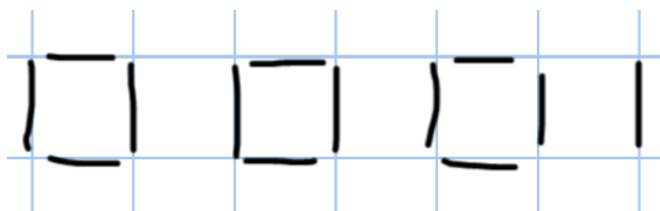
Use of lollipop sticks to form wholes-squares are made because we are dividing by 4.



There are 3 whole squares, with 1 left over.

Pictorial

Children to represent the lollipop sticks pictorially.

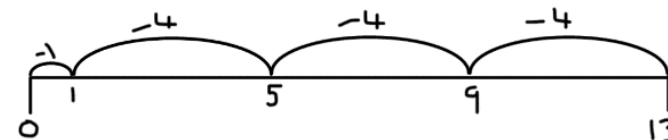


Abstract

$$13 \div 4 = 3 \text{ remainder } 1$$

Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'



Sharing using place value counters. $42 \div 3 = 14$



10s	1s

→

10s	1s
●	
●	
●	

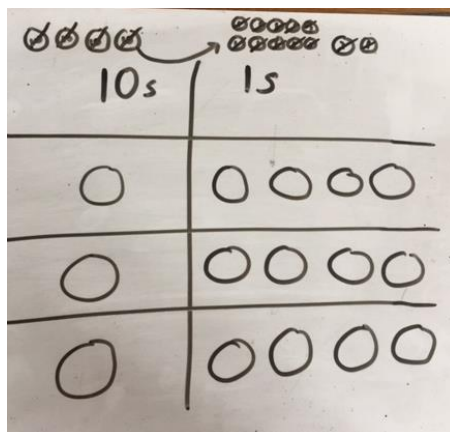


10s	1s
●	●●●●
●	●●●●
●	●●●●

= 14

10s	1s
●	
●	
●	

Children to represent the place value counter pictorially.



Children to be able to make sense of the place value counters and write calculations to show the process.

$$42 \div 3$$

$$42 = 30 + 12$$

$$30 \div 3 = 10$$

$$12 \div 3 = 4$$

$$10 + 4 = 14$$

Years 4, 5 and 6

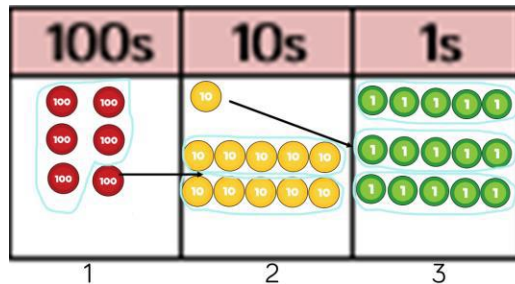
Division

Key language: share, group, divide, divided by, half.

Concrete

Short division using place value counters to group.

$$615 \div 5$$



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Long division using place value counters.

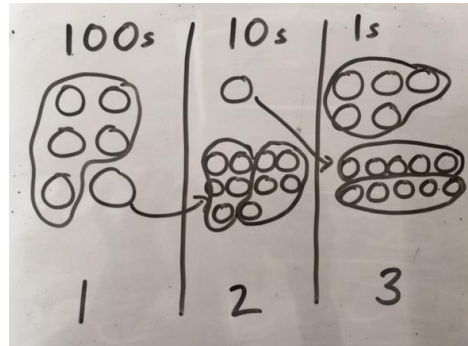
$$2544 \div 12$$



We can't group 2 thousands into groups of 12 so will exchange them.

Pictorial

Represent the place value counters pictorially.



Abstract

Children to the calculation using the short division scaffold.

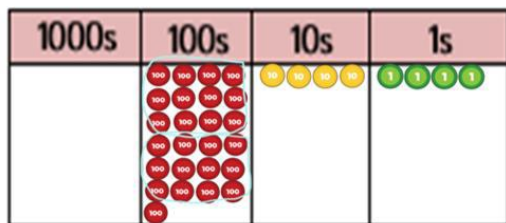
$$\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}$$

Pupils apply this with a remainder

$$\begin{array}{r} 196 \text{ r } 3 \\ 6 \overline{) 115739} \end{array}$$

Then interpret the remainder as a fraction $\frac{3}{6}$ then a decimal: 0.5

$$\begin{array}{r} 196.5 \\ 6 \overline{) 115739.30} \end{array}$$



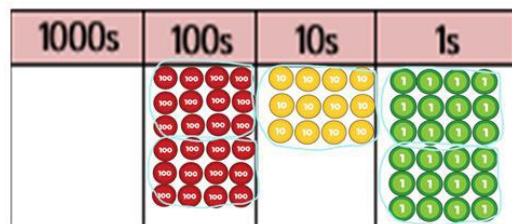
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Progressing to long division to find a decimal remainder:

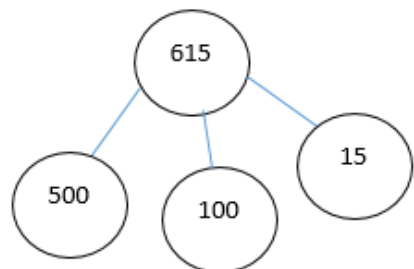
$$\begin{array}{r} 28 \cdot 8 \\ 15 \overline{) 4320} \\ \underline{30} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

Simplify this method into long division:

$$\begin{array}{r} 28.8 \\ 15 \overline{) 4320} \\ \underline{30} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

Division Conceptual Variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



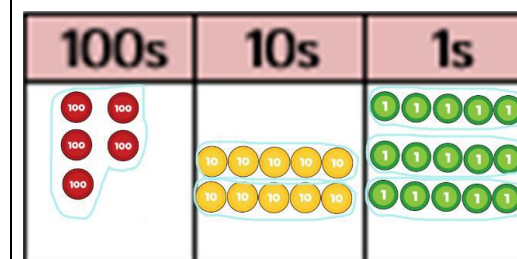
I have £615 and share it equally between 5 bank accounts.
How much will be in each account?
615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{)615}$$

$$615 \div 5 =$$

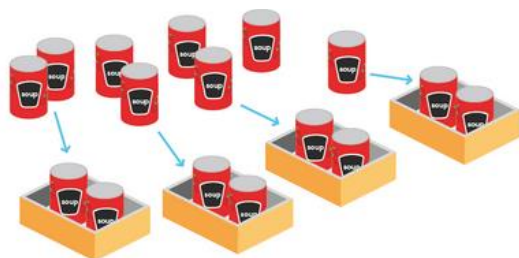
$$\square = 615 \div 5$$

What is the calculation?
What is the answer?



EYFS and Year 1

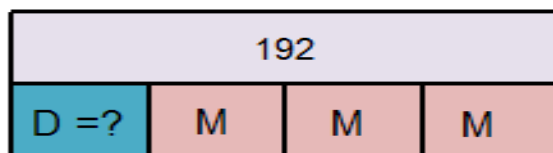
There are 8 cans.



There are 4 boxes of 2 cans.

Year 4

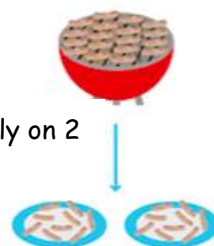
Desmond and Melissa collect cards. They have 192 cards in all. Melissa has three times as many cards as Desmond. How many cards does Desmond have?



Year 2

There are 18 sausages.

Put 18 sausages equally on 2 plates.



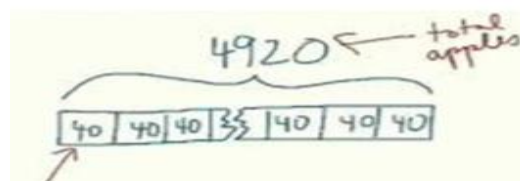
There are 9 sausages on 2 plates: $18 \div 2 = 9$.



Year 5

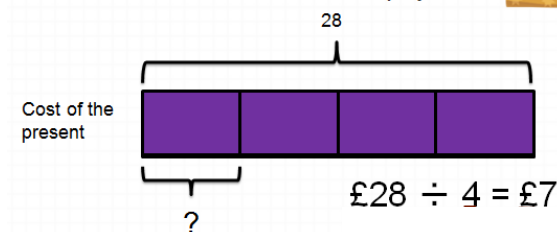
Bar Model to support understanding of problem solving:

Frank has 4920 apples. He needs to put them into baskets of 40. How many baskets does he need?



Year 3

Four children bought a present for £28. They shared the costs equally. How much did each child pay?



Year 6

Paul and David hire a car together at a cost of £297.50. Paul pays 6 times more than David. How much does David pay?

